8.1 Distribution of the Sample Mean

Fill in the blanks to correctly complete the sentence below.

Suppose a simple random sample of size $n$ is drawn from a large population with mean $\mu$ and standard deviation $\sigma$. The sample distribution of $\bar{x}$ has mean $\mu_{\bar{x}} = \underline{\text{______}}$ and standard deviation $\sigma_{\bar{x}} = \underline{\text{______}}$.

Suppose a simple random sample of size $n$ is drawn from a large population with mean $\mu$ and standard deviation $\sigma$. The sample distribution of $\bar{x}$ has mean $\mu_{\bar{x}} = \underline{\mu}$ and standard deviation $\sigma_{\bar{x}} = \underline{\frac{\sigma}{\sqrt{n}}}$.

The standard deviation of the sampling distribution of $\bar{x}$, denoted $\sigma_{\bar{x}}$, is called the standard error of the mean.

The sampling distribution of the sample mean $\bar{x}$ is the probability distribution of all possible values of the random variable $\bar{x}$ computed from a sample of size $n$ from a population with mean $\mu$ and standard deviation $\sigma$. Suppose that a simple random sample of size $n$ is drawn from a large population with mean $\mu$ and standard deviation $\sigma$. The sampling distribution of $\bar{x}$ will have mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. The standard deviation of the sampling distribution of $\bar{x}$, $\sigma_{\bar{x}}$, is called the standard error of the mean.

The distribution of the sample mean, $\bar{x}$, will be normally distributed if the sample is obtained from a population that is normally distributed, regardless of the sample size.

Choose the correct answer below.

True

To cut the standard error of the mean in half, the sample size must be increased by a factor of **four**.
A simple random sample of size $n = 61$ is obtained from a population with $\mu = 78$ and $\sigma = 8$. Does the population need to be normally distributed for the sampling distribution of $\bar{x}$ to be approximately normally distributed? Why?

What is the sampling distribution of $\bar{x}$?

If a random variable, $X$, is normally distributed, the distribution of the sample mean, $\bar{x}$, is normally distributed.

Further, the Central Limit Theorem states that the sampling distribution of the sample mean, $\bar{x}$, becomes approximately normal as the sample size, $n$, increases, regardless of the shape of the underlying population.

Use the information given in the previous steps to determine if the population needs to be normally distributed for the sampling distribution of $\bar{x}$ to be approximately normally distributed.

To determine the sampling distribution of $\bar{x}$, first determine its shape.

- Skewed left
- Normal or approximately normal
- Student's $t$-distribution
- Uniform

Suppose that a simple random sample of size $n$ is drawn from a population with mean $\mu$ and standard deviation $\sigma$. The sampling distribution of $\bar{x}$ has a mean of $\mu_{\bar{x}} = \mu$ and a standard deviation given by the formula below.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

What is the mean of the sampling distribution of $\bar{x}$?

$$\mu_{\bar{x}} = 78$$

Calculate $\sigma_{\bar{x}}$, the standard deviation of the sampling distribution of $\bar{x}$. Begin by substituting values for $\sigma$ and $n$ into the given formula for $\sigma_{\bar{x}}$.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{8}{\sqrt{61}} = 1.024$$

Determine $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ from the given parameters of the population and sample size.

$\mu = 82$, $\sigma = 7$, $n = 49$

$$\mu_{\bar{x}} = 82$$

$$\sigma_{\bar{x}} = \frac{7}{\sqrt{49}} = 1$$
Complete parts (a) through (d) for the sampling distribution of the sample mean shown in the accompanying graph. Click the icon to view the graph.

(a) What is the value of \( \mu_x \)?

The value of \( \mu_x \) is \( 700 \).

(b) What is the value of \( \sigma_x \)?

The value of \( \sigma_x \) is \( 30 \).

(c) If the sample size is \( n = 16 \), what is likely true about the shape of the population?

○ A. The shape of the population is normal.
○ B. The shape of the population is skewed left.
○ C. The shape of the population is skewed right.
○ D. The shape of the population cannot be determined.

(d) If the sample size is \( n = 16 \), what is the standard deviation of the population from which the sample was drawn?

The standard deviation of the population from which the sample was drawn is \( 120 \). \( \frac{30}{4} = \sigma = 120 \)

Suppose a simple random sample of size \( n = 81 \) is obtained from a population with \( \mu = 82 \) and \( \sigma = 9 \).

(a) Describe the sampling distribution of \( \bar{x} \).

(b) What is \( P(\bar{x} > 83.5) \)?

(c) What is \( P(\bar{x} \leq 79.8) \)?

(d) What is \( P(81.2 < \bar{x} < 84.4) \)?

(a) Choose the correct description of the shape of the sampling distribution of \( \bar{x} \).

○ A. The distribution is skewed left.
○ B. The distribution is approximately normal.
○ C. The distribution is uniform.
○ D. The distribution is skewed right.
○ E. The shape of the distribution is unknown

Find the mean and standard deviation of the sampling distribution of \( \bar{x} \).

\[ \mu_x = 82 \]
\[ \sigma_x = 1 \]

(b) \( P(\bar{x} > 83.5) = .0560 \) (Round to four decimal places as needed.)

(c) \( P(\bar{x} \leq 79.8) = .0139 \) (Round to four decimal places as needed.)

(d) \( P(81.2 < \bar{x} < 84.4) = .7799 \) (Round to four decimal places as needed.)
Suppose a simple random sample of size \( n = 46 \) is obtained from a population with \( \mu = 57 \) and \( \sigma = 16 \).

(a) What must be true regarding the distribution of the population in order to use the normal model to compute the sample mean? Assuming the normal model can be used, describe the sampling distribution \( \bar{x} \).

(b) Assuming the normal model can be used, determine \( P(\bar{x} < 70.6) \).

(c) Assuming the normal model can be used, determine \( P(\bar{x} \geq 68.3) \).

(a) What must be true regarding the distribution of the population?

- A. The population must be normally distributed and the sample size must be large.
- B. Since the sample size is large enough, the population distribution does not need to be normal.
- C. The population must be normally distributed.
- D. The sampling distribution must be assumed to be normal.

Assuming the normal model can be used, describe the sampling distribution \( \bar{x} \).

- A. Approximately normal, with \( \mu_{\bar{x}} = 67 \) and \( \sigma_{\bar{x}} = 16 \)
- B. Approximately normal, with \( \mu_{\bar{x}} = 67 \) and \( \sigma_{\bar{x}} = \frac{16}{\sqrt{46}} \)
- C. Approximately normal, with \( \mu_{\bar{x}} = 67 \) and \( \sigma_{\bar{x}} = \frac{46}{\sqrt{15}} = 2.359 \)

(b) \( P(\bar{x} < 70.6) = 0.9365 \) (Round to four decimal places as needed.)

(c) \( P(\bar{x} \geq 68.3) = 0.2908 \) (Round to four decimal places as needed.)

Suppose the lengths of the pregnancies of a certain animal are approximately normally distributed with mean \( \mu = 184 \) days and standard deviation \( \sigma = 18 \) days. Complete parts (a) through (f) below.

Click here to view the standard normal distribution table (page 1).
Click here to view the standard normal distribution table (page 2).

(a) What is the probability that a randomly selected pregnancy lasts less than 177 days?

The probability that a randomly selected pregnancy lasts less than 177 days is approximately 0.3487. (Round to four decimal places as needed.)

Interpret this probability. Select the correct choice below and fill in the answer box within your choice. (Round to the nearest integer as needed.)

- A. If 100 pregnant individuals were selected independently from this population, we would expect 35 pregnancies to last less than 177 days.

(b) Suppose a random sample of 19 pregnancies is obtained. Describe the sampling distribution of the sample mean length of pregnancies.

The sampling distribution of \( \bar{x} \) is normal with \( \mu_{\bar{x}} = 184 \) and \( \sigma_{\bar{x}} = \frac{18}{\sqrt{19}} = 3.079 \).

(c) What is the probability that a random sample of 19 pregnancies has a mean gestation period of 177 days or less?

The probability that the mean of a random sample of 19 pregnancies is less than 177 days is approximately 0.0460. (Round to four decimal places as needed.)
BETWEEN

Add and subtract 10 from the mean

The reading speed of second grade students in a large city is approximately normal, with a mean of 89 words per minute (wpm) and a standard deviation of 10 wpm. Complete parts (a) through (e)

(a) What is the probability a randomly selected student in the city will read more than 93 words per minute?

The probability is 0.3446.
(Round to four decimal places as needed.)

(b) What is the probability that a random sample of 13 second grade students from the city results in a mean reading rate of more than 93 words per minute?

The new Std. Dev. \(= \frac{10}{\sqrt{13}} = 2.77\)

The probability is 0.0746.
(Round to four decimal places as needed.)

(c) What is the probability that a random sample of 26 second grade students from the city results in a mean reading rate of more than 93 words per minute?

The new Std. Dev. \(= \frac{10}{\sqrt{26}} = 1.96\)

The probability is 0.0297.
(Round to four decimal places as needed.)

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

- A. Increasing the sample size increases the probability because \(\sigma_x\) decreases as \(n\) increases.
- B. Increasing the sample size increases the probability because \(\sigma_x\) increases as \(n\) increases.
- C. Increasing the sample size decreases the probability because \(\sigma_x\) decreases as \(n\) increases.

C. If 100 independent random samples of size \(n = 19\) pregnancies were obtained from this population, we would expect 5 sample(s) to have a sample mean of 177 days or less.

C. If 100 independent random samples of size \(n = 36\) pregnancies were obtained from this population, we would expect 1 sample(s) to have a sample mean of exactly 177 days.

(d) What is the probability that a random sample of 36 pregnancies has a mean gestation period of 177 days or less?

The probability that the mean of a random sample of 36 pregnancies is less than 177 days is approximately 0.0098. (Round to four decimal places as needed.)

C. If 100 independent random samples of size \(n = 36\) pregnancies were obtained from this population, we would expect 1 sample(s) to have a sample mean of 177 days or less.

(e) What might you conclude if a random sample of 36 pregnancies resulted in a mean gestation period of 177 days or less?

This result would be unusual, so the sample likely came from a population whose mean gestation period is less than 184 days.

(f) What is the probability a random sample of size 19 will have a mean gestation period within 10 days of the mean?

The probability that a random sample of size 19 will have a mean gestation period within 10 days of the mean is 0.9845. (Round to four decimal places as needed.)

BETWEEN

Add and subtract 10 from the mean
The reading speed of second grade students in a large city is approximately normal, with a mean of 89 words per minute (wpm) and a standard deviation of 10 wpm. Complete parts (a) through (e).

(a) What is the probability a randomly selected student in the city will read more than 93 words per minute?

The probability is 0.3446.
(Round to four decimal places as needed.)

(b) What is the probability that a random sample of 13 second grade students from the city results in a mean reading rate of more than 93 words per minute?

The probability is 0.0746.
(Round to four decimal places as needed.)

(c) What is the probability that a random sample of 26 second grade students from the city results in a mean reading rate of more than 93 words per minute?

The probability is 0.0207.
(Round to four decimal places as needed.)

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

- A. Increasing the sample size increases the probability because \( \sigma_x \) increases as \( n \) increases.
- B. Increasing the sample size increases the probability because \( \sigma_x \) decreases as \( n \) increases.
- C. Increasing the sample size decreases the probability because \( \sigma_x \) decreases as \( n \) increases.
- D. Increasing the sample size decreases the probability because \( \sigma_x \) increases as \( n \) increases.

(e) A teacher instituted a new reading program at school. After 10 weeks in the program, it was found that the mean reading speed of a random sample of 19 second grade students was 91.6 wpm. What might you conclude based on this result? Select the correct choice and fill in the answer box in your choice below.

- A mean reading rate of 91.6 wpm is not unusual since the probability of obtaining a result of 91.6 wpm or more is \( \sigma = \frac{10}{\sqrt{19}} = 2.29 \). STAT – CALC-NORMAL

\[ P(X \geq 91.6) = \frac{0.1285}{0.12850332} \] Since \( 0.1285 > 0.05 \)

13 for every 100

(f) There is a 5% chance that the mean reading speed of a random sample of 21 second grade students will exceed what value?

\[ \sigma = \frac{10}{21} = 2.1822 \]

\[ P(X \geq 95.19) = 0.05 \]

\[ = 95.19 \]
Suppose a geyser has a mean time between eruptions of 88 minutes. If the interval of time between the eruptions is normally distributed with standard deviation 21 minutes, answer the following questions.

(a) What is the probability that a randomly selected time interval between eruptions is longer than 98 minutes?

The probability that a randomly selected time interval is longer than 98 minutes is approximately \[ \frac{3170}{10000} \] (Round to four decimal places as needed.)

(b) What is the probability that a random sample of 8 time intervals between eruptions has a mean longer than 98 minutes?

The probability that the mean of a random sample of 8 time intervals is more than 98 minutes is approximately \[ \frac{685}{1000} \] (Round to four decimal places as needed.)

(c) What is the probability that a random sample of 19 time intervals between eruptions has a mean longer than 98 minutes?

The probability that the mean of a random sample of 19 time intervals is more than 98 minutes is approximately \[ \frac{1019}{100000} \] (Round to four decimal places as needed.)

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result. Choose the correct answer below.

- A. The probability increases because the variability in the sample mean decreases as the sample size increases.
- B. The probability decreases because the variability in the sample mean increases as the sample size increases.
- C. The probability increases because the variability in the sample mean increases as the sample size increases.
- D. The probability decreases because the variability in the sample mean decreases as the sample size increases.

(e) What might you conclude if a random sample of 19 time intervals between eruptions has a mean longer than 98 minutes? Choose the best answer below.

- A. The population mean may be greater than 88.

Consider a random variable \( X \) that is normally distributed. Complete parts (a) through (d) below.

(This is a reading assessment question. Be certain of your answer because you only get one attempt on this question.)

(a) If a random variable \( X \) is normally distributed, what will be the shape of the distribution of the sample mean?

- Normal
- Skewed right
- Skewed left
- Cannot be determined

(b) If the mean of a random variable \( X \) is 35, what will be the mean of the sampling distribution of the sample mean?

\[ \mu_X = 35 \]

(c) As the sample size \( n \) increases, what happens to the standard error of the mean?

C. The standard error of the mean decreases.

(d) If the standard deviation of a random variable \( X \) is 25 and a random sample of size \( n = 17 \) is obtained, what is the standard deviation of the sampling distribution of the sample mean?

\[ \sigma_X = \frac{25}{\sqrt{17}} \] (Type an exact answer, using radicals as needed.)